What Makes a Circle a Circle?

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Circles – features

- Always curved
- Never ends
- “center” and “radius”
- Angles
- Diameter
- Tangent

- \( \frac{Circumference}{Diameter} = \pi \approx 3 \)
- Rolling a circle on its edge
- Area = \( \pi r^2 \)
  - Area = \( \frac{\pi}{4} d^2 \)
  - Area \( \approx \frac{3}{4} d^2 \)
How many squares make a circle?

Area of square and area of circle
More properties, please!
Circle and areas of plane figures
Circle is the most fat of all!

- Isoperimetric inequality
  - If a plane figure has perimeter $C$ and area $A$, then $4\pi A \leq C^2$
  - If this is equal, then we have a circle!
Maybe someone mentions Chords in a circle

☐ Angles?
☐ Lengths?

$AB = 3.00$ in.
Rolling out a rope

- Circular section?
- If a plane figure has constant diameter, is it a circle?
- What do we mean by “diameter” or “width?”
Support Line

- For finite set, or compact planar figure.
- Line defines a half-plane.
- Figure is on one side of the line. Figure touches the line.
- Try with our pictures.
Now draw a 2\textsuperscript{nd} line, parallel

- Move it until it becomes a support line on the other side.
“Width” – distance between support lines
What happens?

- Minimum “width”?  
- Maximum width?  
- Width as a function of the angle? (trigonometry if you like it)  
- Graph of width as a function of angle?  
- Other figures? E.g., See Warm-up figures.  
- Constant width?
Constant width

- What about a circle?
- If a figure has constant width must it be a circle?
- Major exploration time.
Constructing a Reuleaux Triangle.
Calculate

- Width of Reuleaux Triangle
- In every direction.
- “Constant width”
Rolling a Reuleaux triangle.

- Now what do you think about making a rope out of clay?
Inside a square.

- A square touches a circle all around. And the circle can rotate inside the square touching the sides.

- Does any other figure do that?
Reuleaux triangle encased by a square.

- But what happens as it rotates?
- “Drilling a square hole”
Circumference

- Circumference of a circle is $\pi d$
- If a figure has circumference $\pi d$, must it be a circle?
Reuleaux Triangle, with width d.

- Figure out its circumference.
Reuleaux Triangle, with width $d$.

- Figure out its circumference.
  
  Each arc has length $\frac{1}{6} 2\pi d$.
  Three arcs.
  Circumference is $\pi d$.
  Just like a circle!
What shapes could be a man-hole cover? What about the piano-mover problem?
How about the area of a circle?  
Of a Reuleaux triangle.

**Circle**
- $A = \pi r^2$
- We can demonstrate!
- $A = \frac{\pi}{4} d^2$

**Reuleaux Triangle**
- Aha! Interesting problem
- Area of sector = 1/6 area of circle = $\frac{1}{6} \pi d^2$
- Area of triangle = $\frac{1}{2} \text{base} \times \text{alt} = \frac{1}{2} d \frac{d\sqrt{3}}{2} = \frac{\sqrt{3}}{4} d^2$
- Area of Reuleaux triangle = 3 sectors – 2 triangles = $\frac{\pi}{2} d^2 - \frac{\sqrt{3}}{2} d^2$
Every good problem leads to a new problem

- Can you avoid the “corners” on the Reuleaux triangle?
- What about more sides? For example, the pentagon?