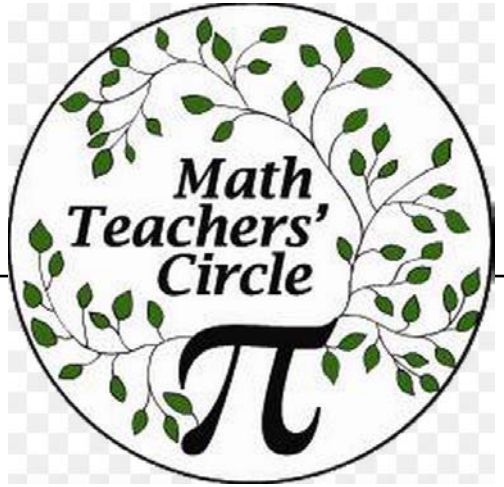
What Makes a Circle a Circle?



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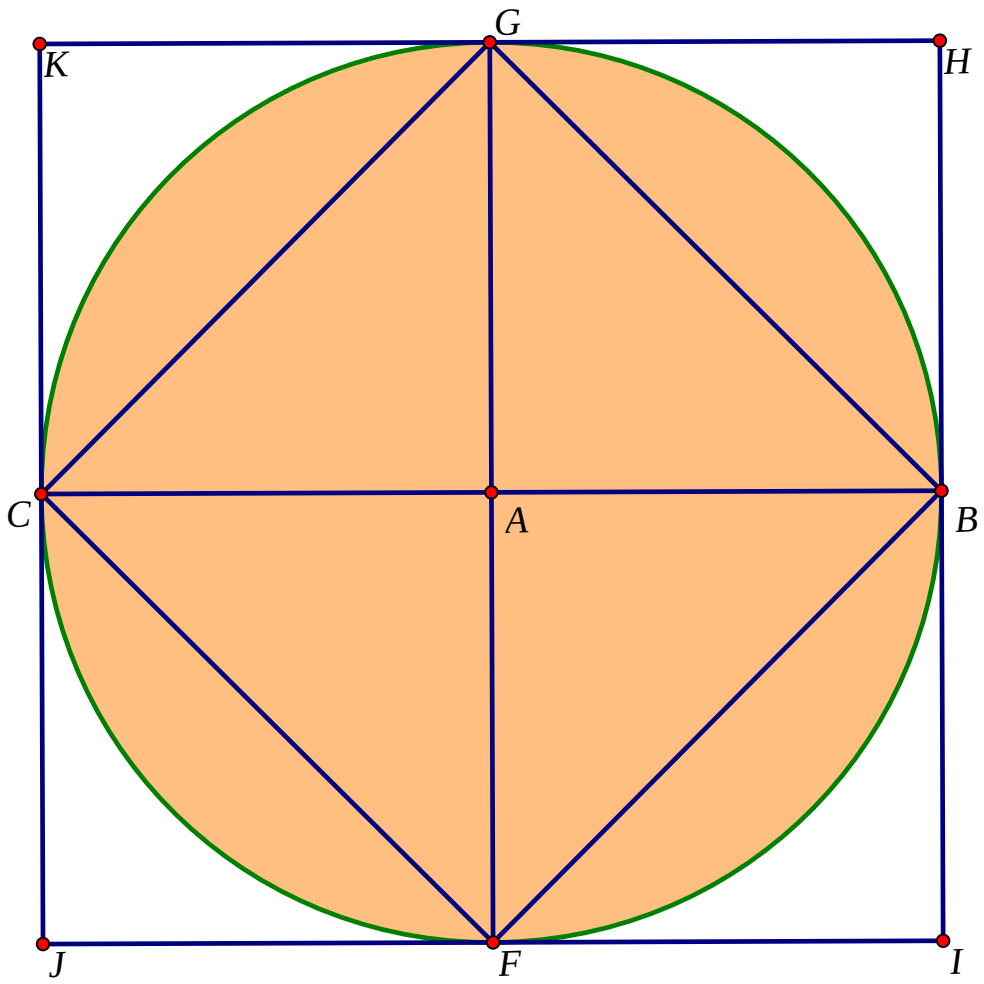
Circles – features

- Always curved
- Never ends
- “center” and “radius”
- Angles
- Diameter
- Tangent

- $\frac{Circumference}{Diameter} = \pi \approx 3$
- Rolling a circle on its edge
- Area = πr^2
  - Area = $\frac{\pi}{4} d^2$
  - Area $\approx \frac{3}{4} d^2$

Area of square and area of circle

How many squares make a circle?



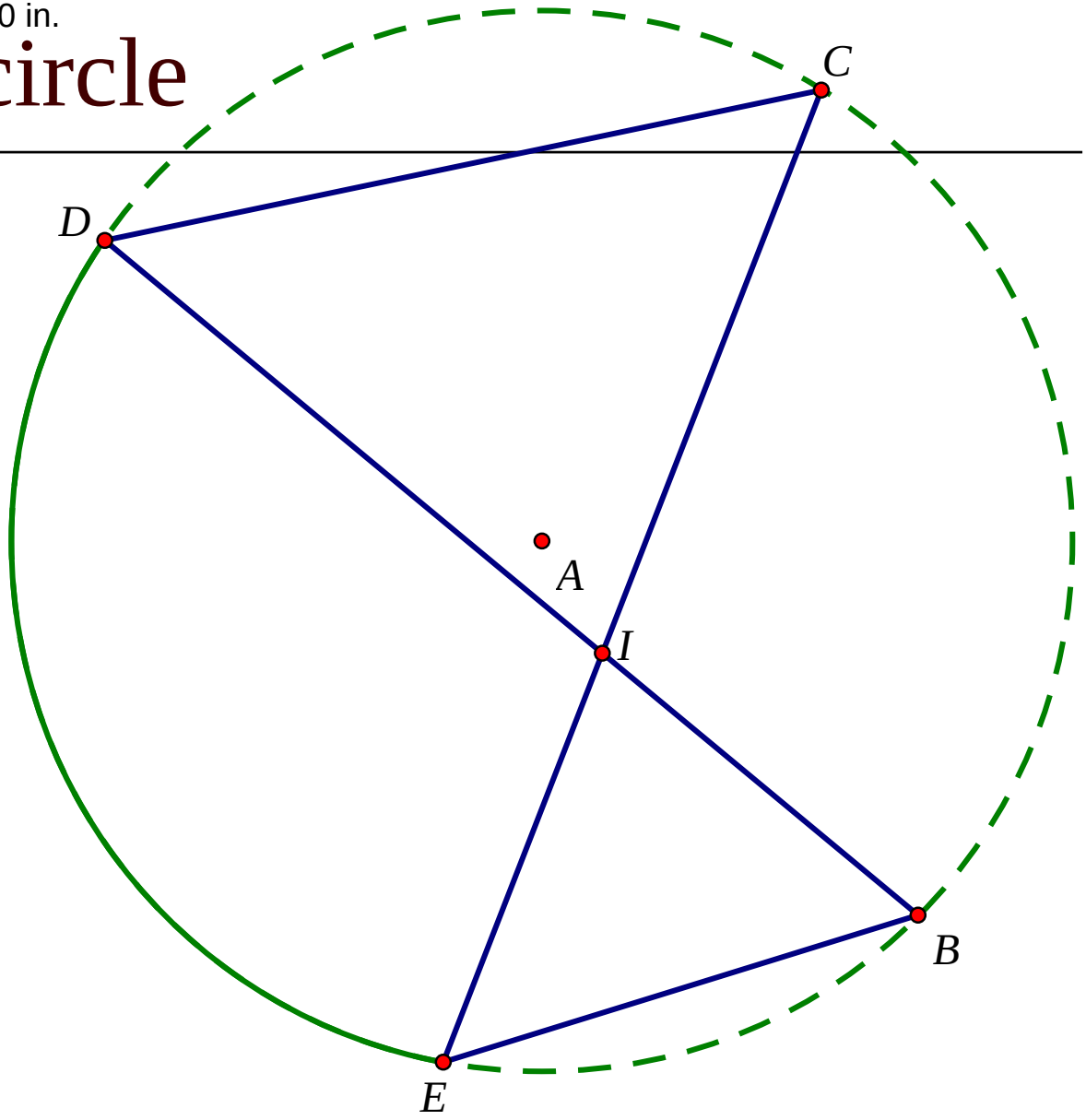
More properties, please!

Circle and areas of plane figures
Circle is the most fat of all!

- Isoperimetric inequality
  - If a plane figure has perimeter C and area A, then $4\pi A \leq C^2$
  - If this is equal, then we have a circle!

Maybe someone mentions Chords in a circle

- Angles?
- Lengths?



Rolling out a rope

- Circular section?

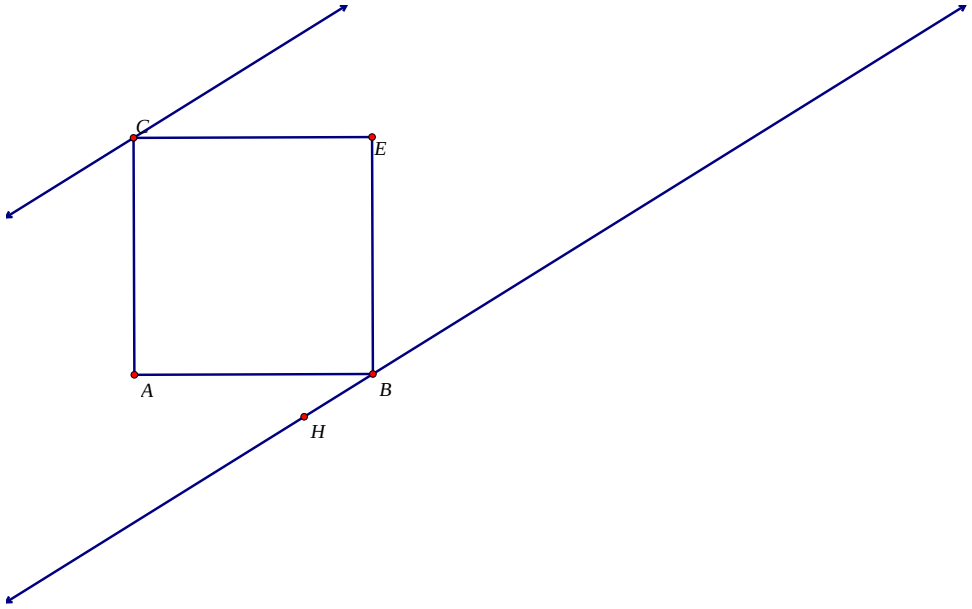
- If a plane figure has constant diameter, is it a circle?
- What do we mean by “diameter” or “width?”

Support Line

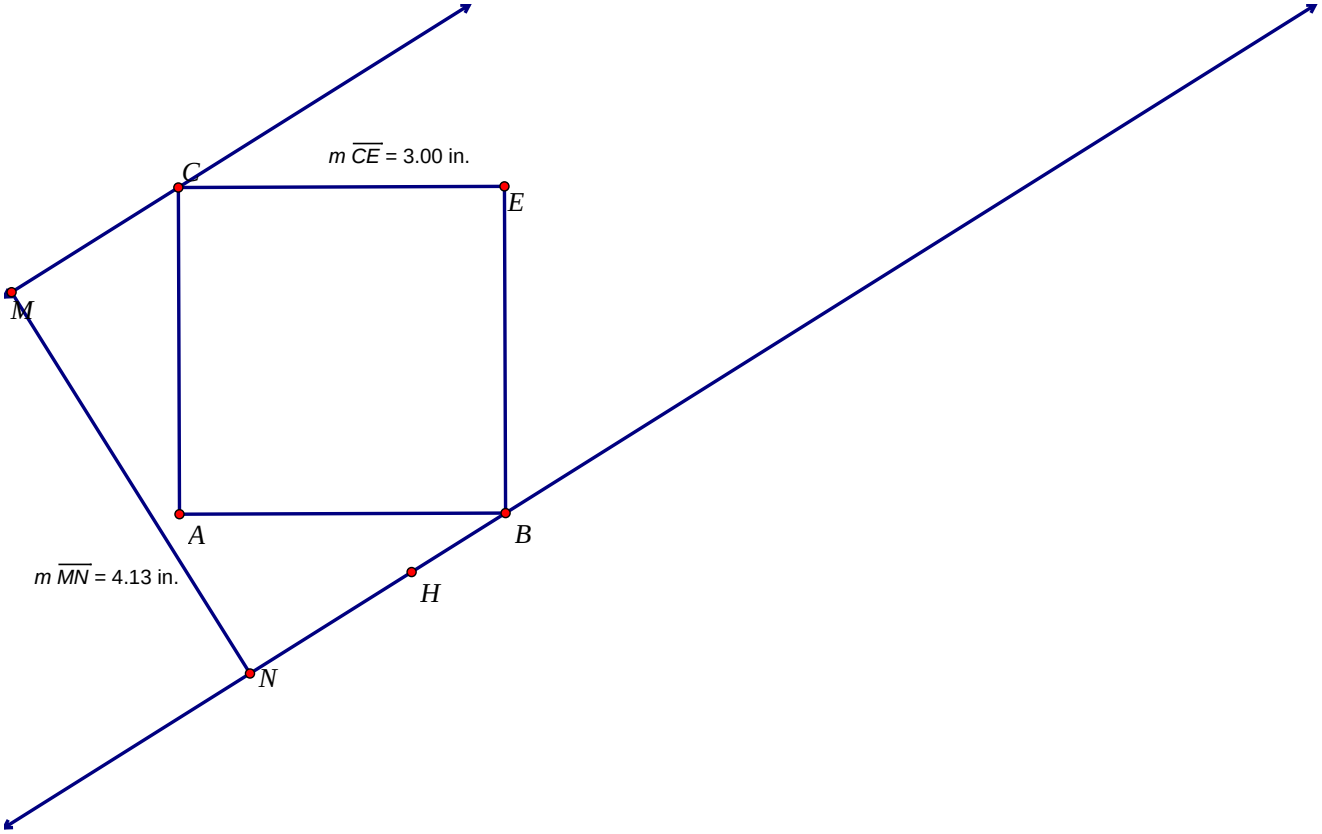
- For finite set, or compact planar figure.
- Line defines a half-plane.
- Figure is on one side of the line. Figure touches the line.
- Try with our pictures.

Now draw a 2nd line, parallel

- Move it until it becomes a support line on the other side.



“Width” – distance between support lines



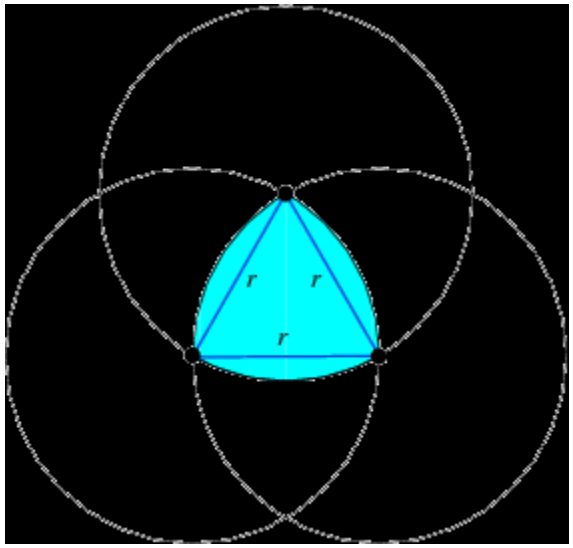
What happens?

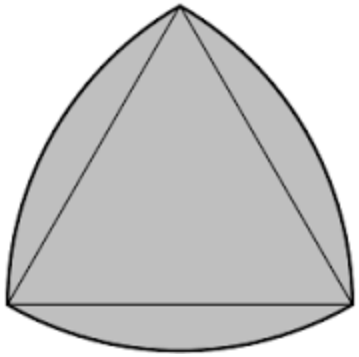
- Minimum “width”?
- Maximum width?
- Width as a function of the angle? (trigonometry if you like it)
- Graph of width as a function of angle?
- **Other figures**? E.g., See Warm-up figures.
- Constant width?

Constant width

- What about a circle?
- If a figure has constant width must it be a circle?
- Major exploration time.

Constructing a Reuleaux Triangle.





Calculate

- Width of Reuleaux Triangle
- In every direction.
- “Constant width”

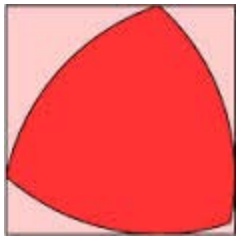
Rolling a Reuleaux triangle.

- Now what do you think about making a rope out of clay?

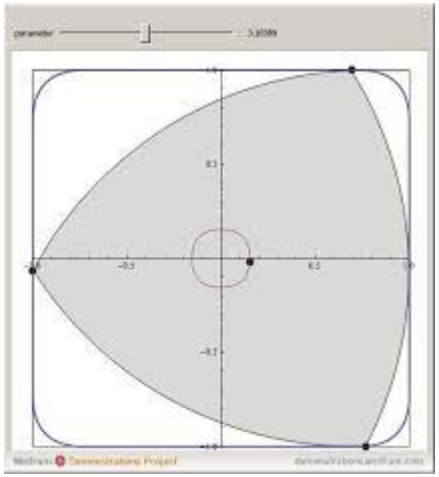
Inside a square.

- A square touches a circle all around. And the circle can rotate inside the square touching the sides.
- Does any other figure do that?

Reuleaux triangle encased by a square.



- But what happens as it rotates?
- “Drilling a square hole”

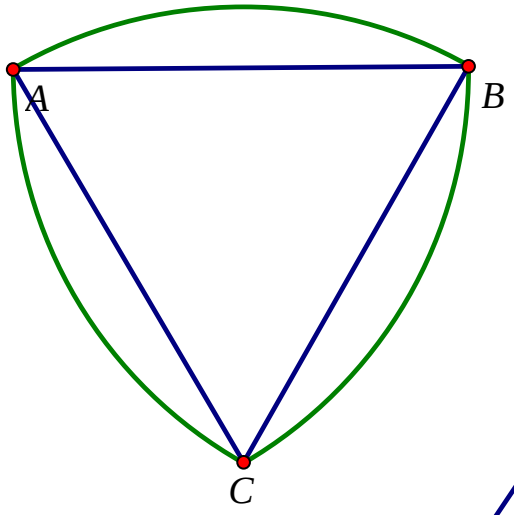


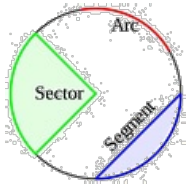
Circumference

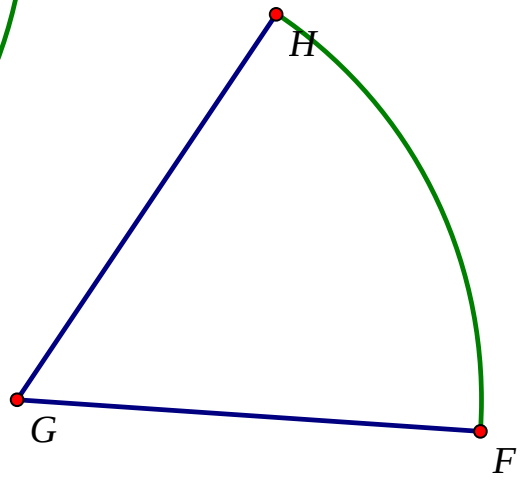
- Circumference of a circle is πd
- If a figure has circumference πd, must it be a circle?

Reuleaux Triangle, with width d.

- Figure out its circumference.

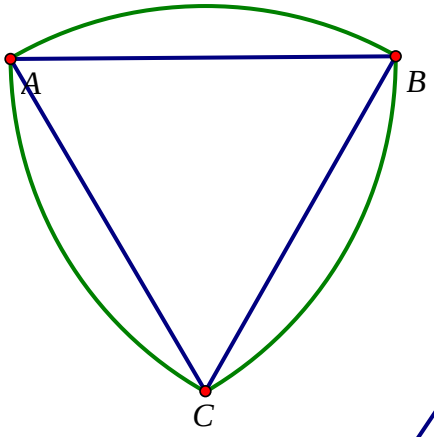


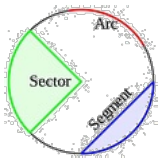


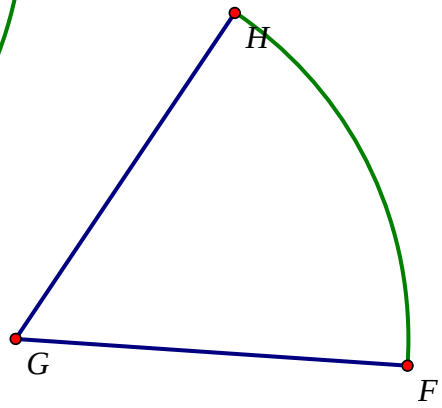


Reuleaux Triangle, with width d.

- Figure out its circumference.







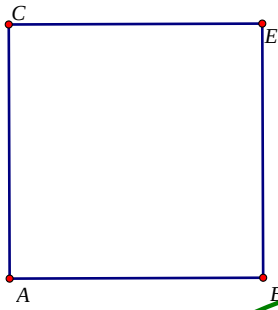
Each arc has length $\frac{1}{6}2\pi d$.

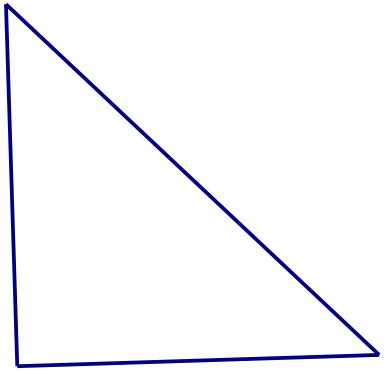
Three arcs.

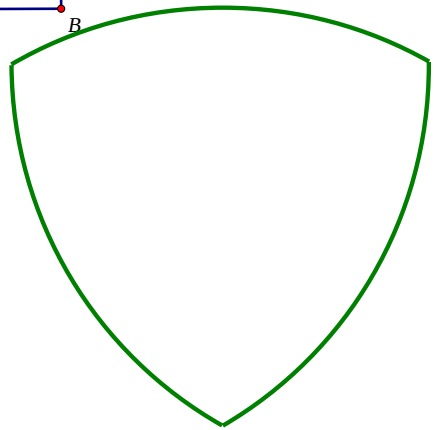
Circumference is πd.

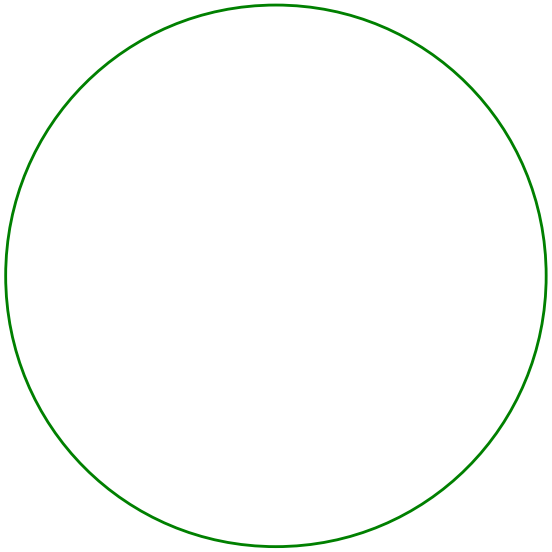
Just like a circle!

What shapes could be a man-hole cover? What about the piano-mover problem?





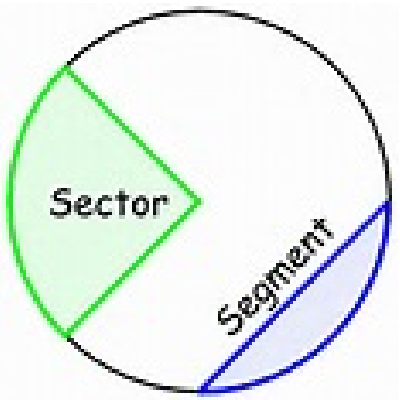




How about the area of a circle? Of a Reuleaux triangle.

Circle

- A = πr^2
- We can demonstrate!
- $A = \frac{\pi}{4} d^2$



Reuleaux Triangle

Aha! Interesting problem

Area of sector = 1/6 area of circle = $\frac{1}{6}\pi d^2$

Area of triangle = ½ base* alt = $\frac{1}{2} d \frac{d\sqrt{3}}{2} = \frac{\sqrt{3}}{4} d^2$

Area of Reuleax triangle = 3 sectors – 2 triangles = $\frac{\pi}{2} d^2 - \frac{\sqrt{3}}{2} d^2$

Every good problem leads to a new problem

- Can you avoid the “corners” on the Reuleaux triangle?
- What about more sides? For example, the pentagon?