

FROGS AND TOADS

AN INTRODUCTION TO MATH CIRCLE STYLE PROBLEM SOLVING
(OR AT LEAST TO WHAT I MEAN BY THAT)

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Thanks for the chance to present! So, I'm with the math teachers' circle at Loyola University Chicago. The main goal of our circle is to engage teachers in open ended, interesting, problem solving. Today I'm going to talk about the problems I use to introduce what that means. I've used these a number of times, with teachers, pre-service teachers, other math majors, and even professors. But today is short, so I'm going to talk about where I'm going before introducing the questions.

I'm trying to engage teachers with good problems, to help them develop (and hopefully then teach) good problem solving strategies/mindset. But first, what is a good problem? For me it is a problem where

- When people first look at it, they **do not know what to do**.
- Solving it involves some **exploration**, meaning doing things to see what will happen, not expecting them to lead directly to the answer.

You'll see more what I mean when I introduce the problems.

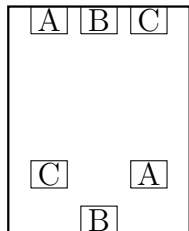
Even more importantly though, what is good problem solving mindset? I've been led more and more to the language I hear from my 5 year old. Her preschool has been doing a unit called I Can Problem Solve (ICPS). Well, problem solving! In that context it really means "don't get in fights with the other 5 year olds," but that is a kind of problem solving. Anyway, I'm talking about the following advice:

- You need to listen to the question. It has things to say, and they might be interesting. If you are always the one talking, you will miss out!
- You should be willing to compromise with the problem. Maybe you won't solve it as stated. Maybe you should do an easier problem first. Maybe you'll have an idea for a similar problem that works out better, and you can learn from that. You'll have more fun if you can be flexible!
- Sometimes the problem wins, and that is ok! It isn't about who wins, its about how much fun you have together!
- You can keep playing even after the problem is solved! [with extensions, generalizations, etc.]

I usually talk about these things after first having people do the following problem (which I have them do individually, and give only 3-5 minutes): Connect A to A, B to B, and C to C below, with lines

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inside the box, and no crossing lines. My students once called this the “think outside the box but stay inside the box” question.

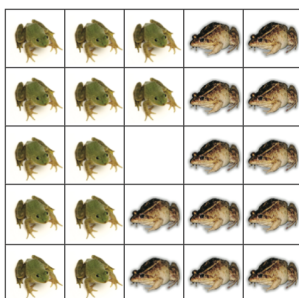


I love this because none of the “formulaic” problem solving techniques help at all. Draw a picture...already done. Make an equation...?? Organize information...it looks pretty organized. Look for patterns? That actually can help if you are creative enough, but maybe it should be in the good problem solving column anyway.

But if you just start trying things and **listen to the problem** by thinking about what goes wrong, you will definitely solve it. If you **compromise** by simplifying somehow, say by first deleting the Cs, you will solve it (there are actually many interesting ways to simplify this question). In the end, if you’ve really understood things, you should be able to see what happens if, for example, I add a D box, or even an E and an F. That is, it is worth while to **keep playing** even if you have a solution.

Then I do the frog and toad problem below. I got this from the summer 2013 MTCircular magazine, where (with slightly altered wording) it was the problem circle problem contributed by Joshua Zucker:

In this game, frogs can move right or down. Toads can move left or up. A frog or toad can slide into an adjacent empty space. They can also jump over exactly one of the other kind of animal to land in an empty space. They want to swap places (so that every place that now has a frog will have a toad, and vice-versa). Can it be done? How? ...



I have them do this in groups of about 4, and plan on it taking an hour or so. Notice: this is clearly not from the curriculum. That is a point I make: the solution in a sense doesn’t matter, it is experiencing

finding the solution that is worthwhile. It isn't immediately clear to anyone what to do (or even if it is possible), but you can start doing things...very quickly though, you see it is too complicated. You need to do a simpler problem! Most people choose to do a 3 by 3 version of the question, but this step takes longer than you might think...

From there there are three quite distinct ways to solve the problem, each of which gives insight into problem solving:

Method 1: Try and fail to do the 3 by 3 case a bunch of times. Keep a record of the stuck positions. Notice that most of these have three frogs at the bottom or three toads at the top, which is bad (not quite all have this property). Conclude: moving up or down is dangerous...which you should have known! They mostly need to go left-right, not up-down! Systematically go left-right as much as possible. Now you'll solve it...

Method 2: Solve the 3 by 3 case by trial and error, then organize your solution so you can describe it nicely in words, at which point it should generalize. This can be problematic, because there are solutions that don't generalize well...but that just gives a chance to talk about the value to looking for more/better solutions once a problem is solved. It also gives a way to understand what a better solution is: it is one that is easier to explain and generalize.

Method 3: Be more creative about generalizing the problem! It makes sense e.g. for a 3 by 5 rectangle....or 1 by 3! 1 by 3 is easy. 1 by 5 a bit harder, but you'll get it...and now you can notice, in the original 5 by 5 question, you can start by flipping the middle row using the 1 by 5 case! Then move one animal up or down and flip another row. Now you'll get it...

Method 3 is my favorite, and I push some people towards it a bit. But I mostly try to let people go their own course. Because actually my favorite thing is that there are so many nice solutions, and that you can have great discussions about them.

Of course there are extensions for people who finish early (not many do in my experience). You can ask how many moves it takes (I know an answer for my solution, but not a proof that all solutions will have the same number of steps, although I think they do). You can do different rectangles. You can even think about what to do if one of the lengths is even (so the initial hole has to be a bit off-center). I don't know the full answer to that either. So, there is plenty to do!